THE 2009–2010 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box, or more than one box, is marked. Zero points are given for an incorrect answer. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

90 MINUTES

1. Let l be a line whose slope is equal to its y-intercept. There is one point in the x-y plane through which l must pass. What are the coordinates of this point?

   (A) (1, 0)

7. Containers A, B, and C are partially filled with water. If we pour 3 quarts from B into A, A will contain as much water as will B and C combined. This is not done, and instead we pour 5 quarts from A into C. Now C contains as much as B, while A contains 2 quarts more than B. How many quarts of water did container A contain originally?

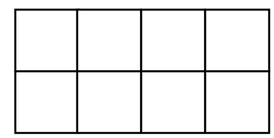
(A) 27 (B) 26 (C) 25 (D) 24 (E) 23

8. Order sin1, sin2, sin3 from smallest to largest (the angles are measured in radians).

(A) sin1 < sin2 < sin3 (B) sin3 < sin2 < sin1, (C) sin1 < sin3 < sin2,

(D) sin2 < sin1< sin3, (E) sin3 < sin1 < sin2

9. The rectangle shown is subdivided into 8 small squares. In how many ways is it possible to shade exactly one-quarter of the rectangle so that <u>both</u> of the following conditions are satisfied?



I. Only whole squares are shaded and
II. No two shaded squares have a common side.

(A) 18 (B) 20 (C) 24 (D) 30 (C) 36

10. Two trains take 3 seconds to clear each other when passing in opposite directions, and 35 seconds when passing in the same direction. If each train travels at a constant rate, compute the ratio of the speed of the faster train to the speed of the slower train.

(A) $\frac{19}{16}$ (B) $\frac{21}{17}$ (C) $\frac{22}{13}$ (D) $\frac{27}{19}$ (E) $\frac{31}{26}$

11. Let K be the set consisting of the first 11 positive integers. A subset of three distinct positive integers is said to be *acceptable* if it contains at most one odd element. If a subset of three different numbers is chosen at random from K, what is the probability that the subset will be *acceptable*?

(A) $\frac{4}{11}$ (B) $\frac{9}{11}$ (C) $\frac{2}{33}$ (D) $\frac{14}{33}$ (E) $\frac{19}{33}$

12. Two numbers are written in base *a* as 32 and 24. The same two numbers are written in base *b* as 43 and 33, respectively. What is the sum of these two numbers in base 10?

(A) 33 (B) 34 (C) 35 (D) 39 (E) 41

20. The triangular numbers, 1, 3, 6, 10, ... are so named because they can be represented geometrically as

How many of the first 100 triangular numbers end with a 0?

SOLUTIONS – KSU MATHEMATICS COMPETITION – 2009

2\.
B   Termr20 Td l zeros ... UKV

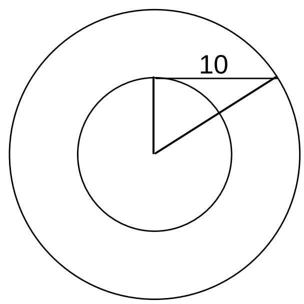
9.

4\.
C   Let d = 100h +10t + u - (100u + 10t + h) = 99(h - u). Then the difference between the three-digit number and the number with its digits reversed is a multiple of 99. Examining the multiples of 99, only **594** has a units digit of 4.

5\.

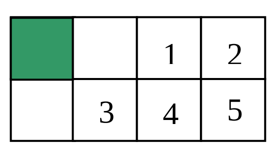
E   Draw radii AP and AQ. Using the Pythagorean Theorem on right triangle APQ, $(QA)^2 – (PA)^2 = 100$. Since the area of the large circle is $\pi(QA)^2$ and the area of the smaller circle is $\pi(PA)^2$, the area of the region between the two circles is

$$\pi(QA)^2 – \pi(PA)^2 = \pi[(QA)^2 – (PA)^2] = \mathbf{100\pi}.$$



6\. C   There a   C= B –

5 = (A – 7) =sin3 < sin1 < sin2.   9. A   Shading one-quarter of the rectangle means shadi



14. C Since BC // AD, $\angle DAP \cong \angle PAB \cong \angle APB$, making ΔAPB an isosceles right triangle. Without loss of generality, let AP = 4 and PD = 3. Then, AB = CD = $2\sqrt{2}$. Using the Pythagorean Theorem on right triangle PCD, PC = 1. Therefore, the ratio of PD to PC is **3:1**.

15. C Let the roots be a, b, and c with a + b = 4. Then, abc = 9 and ab + ac + bc = 12. The second equation can be rewritten as ab + c(a+b) = 9/c + 4c = 12, or $4c^2 - 12c + 9 = (2c - 3)^2 = 0$. So, the last root is $c = \frac{3}{2}$.

20. B The n^{th} triangular number can be represented as $\frac{n(n+1)}{2}$. For a triangular number to end in a zero, the number n(n+1) must be a multiple of 20. Therefore one of n and n+1 must be a multiple of 5 and one (maybe the same one) must be a multiple of 4. The possible values are: 4,15,19,20,24,35,39,40,44,55,59,60,64,75,79,80,84,95,99,100. Therefore, there are **20** triangular numbers that end in zero.

21. A For any one person, the information given can be translated into

$$5n + 10(40-n-q) + 25q = 500 \quad \text{which becomes} \quad 3q-n = 20.$$

Make a chart of all the possibilities.

n	**1**	4	7	10	13	16	19	**22**	25
d	**32**	28	24	20	16	12	8	**4**	0
q	**7**	8	9	10	11	12	13	**14**	15

Since Debbie has twice as many quarters as Don, we need to look for two columns in which the number of quarters in one is twice that in another. The two columns are in bold print. Therefore, Don has **1** nickel.

22. C Using AC as the base, the area of triangle ABC is $\frac{1}{2}(8)(1) = 4$.

Since the area of right triangle ABN = $\frac{1}{2}$

24. C The array

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array} \quad \text{can be written as} \quad \begin{array}{ccc} a & b & c \\ d & a+19 & d+19 \\ g & g-19 & b-19 \end{array}$$

Therefore:

$2a + b + d = 58$ (1)

$a + b + c + d = 56$ (2)

$a + d + 2g = 86$ (3)

$a + b + d + g = 83$ (4)

Now express all variables in terms of b.

Subtracting (4) from (3) gives $g - b = 3$ from which $h = b - 16$.

Subtracting (2) from (4) gives $g - c = 27$ from which $c = b - 24$.

Subtracting (4) from (1) gives $a - g = -25$ from which $a = b - 22$, and $e = b - 3$.

From (1) $2b - 44 + b + d = 58$ or $d = 102 - 3b$ and then $f = 121 - 3b$.

The array

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array} \quad \text{now becomes} \quad \begin{array}{ccc} b-22 & b & b-24 \\ 102-3b & b-3 & 121-3b \\ b+3 & b-16 & b-19 \end{array}$$

Note, b is a positive integer and $b - 24 > 0$. Thus $b \ge 25$.

Also $102 - 3b > 0$. Thus $b \le 33$. Therefore, $b = 25, 26, 27, \ldots, 33$.

So nine different arrays can be formed. For any of them,

$a + e + f + g = (b - 22) + (b - 3) + (121 - 3b) + (b + 3) = 99$.

25. D We are given $f(n) = \left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{n}{3} \right\rfloor$. Suppose $f(n) = k > 0$. If $n = 6k$,

$f(n) = \left\lfloor \frac{6k}{2} \right\rfloor - \left\lfloor \frac{6k}{3} \right\rfloor = \lfloor 3k \rfloor - \lfloor 2k \rfloor = 3k - 2k = k$.

A place to look for other values is at or near 6kl k i Q